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Implementing Gauss's law in Yang-Mills theory and QCD

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Abstract

We construct a transformation that transforms perturbative states into states that implement Gauss's law for 'pure gluonic' Yang-Mills theory and QCD. The fact that this transformation is not and cannot be unitary has special significance. Previous work has shown that only states that are unitarily equivalent to perturbative states necessarily give the same S-matrix elements as are obtained with Feynman rules.

Keywords: Yang-Mills, QCD, Quantum Chromodynamics, non-Abelian, Gauss's law.

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In earlier work, one of us (KH) quantized Yang-Mills theory in the temporal ($A_0^a = 0$) gauge and formulated the constraint that implements Gauss's law by selecting an appropriate subspace for the dynamical time evolution of state vectors [1]. One objective of Ref. [1] was to compare the implications of Gauss's law when it is imposed on the non-Abelian Yang-Mills (YM) theory and Quantum Chromodynamics (QCD) with its role in QED. Despite great similarities between the Abelian and the non-Abelian theories, the inclusion of gauge fields in the non-Abelian charge density is responsible for important differences between QED and QCD [1,2]. Not only is it far more difficult to construct states that implement Gauss's law in non-Abelian gauge theories than in QED; it is also much more important to use states that implement Gauss's law in evaluating S-matrix elements in YM theory and QCD than it is in QED.

In gauge theories, it is standard practice to use Feynman rules in perturbative calculations; these rules implicitly use charged particle states that do *not* obey Gauss's law. In the evaluation of S-matrix elements in QED, perturbative states that do not implement Gauss's law may be safely substituted for states that do implement it. This has been shown to be due to the fact that unitary transformations suffice to construct the latter states from the former [2,3]. But this unitary equivalence does not extend to the non-Abelian gauge theories. The validity of perturbative calculations based on Dirac spinor quarks and free gluons may therefore require qualifications in YM theory and QCD that are not needed in QED. Furthermore, the question has been raised whether the proper implementation of Gauss's law in non-Abelian gauge theories might have significant implications for the confinement of colored states and the conjectured requirement that only color singlets can be asymptotic scattering states [1].

In this paper, we construct states that obey the non-Abelian Gauss's law in 'pure gluonic' YM theory and QCD. Our program is based on the construction of a transformation $\mathcal{T}$ (which must be non-unitary) that transforms perturbative states $|a\rangle$ — in the first instance the perturbative vacuum state $|0\rangle$ — into states that satisfy Gauss's law, and that continue to satisfy Gauss's law even after dynamical time evolution. Unlike James and Landshoff, who had to require matrix elements of a non-terminating progression of powers of the "Gauss's law operator", $J_0^a - \partial_i E_i^a$, to vanish in order to obtain states that implement Gauss's law [4], we find that we do not need to apply progressively escalating powers of projection operators to achieve our objective.

As in Refs. [1,2], we represent the transverse gauge fields and their adjoint momenta as

$$A_{Ti}^a(\mathbf{r}) = \sum_{\mathbf{k};s=1,2} \frac{\epsilon_i^s(\mathbf{k})}{\sqrt{2k}} \left[a_s^a(\mathbf{k})e^{+i\mathbf{k}\cdot\mathbf{r}} + a_s^{a\dagger}(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{r}}\right] , \tag{1}$$

and

$$\Pi_{Ti}^a(\mathbf{r}) = \sum_{\mathbf{k};s=1,2} -i\epsilon_i^s(\mathbf{k})\sqrt{k/2}\left[a_s^a(\mathbf{k})e^{+i\mathbf{k}\cdot\mathbf{r}} - a_s^{a\dagger}(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{r}}\right] , \tag{2}$$

where $a_s^{a\dagger}(\mathbf{k})$ and $a_s^a(\mathbf{k})$ represent 'standard' creation and annihilation operators, respectively, for gluons (or — with the Lie group indices removed — photons) of helicity s. The longitudinal fields are represented in terms of ghost excitation operators, in the form

$$A^a_{Li}(\mathbf{r}) = \sum_{\mathbf{k}} \frac{k_i}{2k^{\frac{3}{2}}} \left[a^a_R(\mathbf{k})e^{+i\mathbf{k}\cdot\mathbf{r}} + a^{a\star}_R(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{r}}\right] , \tag{3}$$

and

$$\Pi^a_{Li}(\mathbf{r}) = \sum_{\mathbf{k}} \frac{-ik_i}{\sqrt{k}} \left[a^a_Q(\mathbf{k})e^{+i\mathbf{k}\cdot\mathbf{r}} - a^{a\star}_Q(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{r}}\right] . \tag{4}$$

The ghost excitation operators obey the commutation rules $[a^a_Q(\mathbf{k}), a^{b\star}_R(\mathbf{k}')] = \delta_{ab}\delta_{\mathbf{k},\mathbf{k}'}$, and $[a^a_R(\mathbf{k}), a^{b\star}_Q(\mathbf{k}')] = \delta_{ab}\delta_{\mathbf{k},\mathbf{k}'}$, with all other commutators vanishing. We define the "Gauss's law operator" $\mathscr{G}^a(\mathbf{r})$

$$\mathscr{G}^a(\mathbf{r}) = \partial_i\Pi^a_i(\mathbf{r}) + J^a_0(\mathbf{r}) = -\partial_i E^a_i(\mathbf{r}) + J^a_0(\mathbf{r}) ; \tag{5}$$

where $J^a_0(\mathbf{r}) = g\, f^{abc} A^b_i(\mathbf{r})\, \Pi^c_i(\mathbf{r})$. $\mathscr{G}^a(\mathbf{r})$ can also conveniently be represented in the form

$$\mathscr{G}^a(\mathbf{r}) = \frac{1}{2}\sum_{\mathbf{k}} \left[\Omega^a(\mathbf{k}) + \Omega^{a\star}(-\mathbf{k})\right] e^{+i\mathbf{k}\cdot\mathbf{r}} , \tag{6}$$

where[1]

$$\Omega^a(\mathbf{k}) = 2k^{\frac{3}{2}} a^a_Q(\mathbf{k}) + J^a_0(\mathbf{k}) . \tag{7}$$

In earlier work [1,2], it was demonstrated that Gauss's law and the gauge choice, $A^a_0 = 0$, could be implemented by imposing

$$\Omega^a(\mathbf{k})|\nu\rangle = 0 \tag{8}$$

on a set of states, $\{|\nu\rangle\}$. $\Omega^a(\mathbf{k})$ — and its adjoint $\Omega^{a\star}(\mathbf{k})$ — commute with the Hamiltonians for YM theory and QCD, as well as (with the Lie group index a removed) for QED. Gauss's law, once imposed by this method, therefore is unaffected by time evolution and remains permanently intact.

In QED, the Noether current commutes with the gauge field, because the photons couple to, but do not carry the electric charge. $\Omega(\mathbf{k})$ therefore commutes with $\Omega^\star(\mathbf{k})$, mirroring the commutation rule between $a_Q(\mathbf{k})$ and $a^\star_Q(\mathbf{k})$. That makes it possible to establish a unitary equivalence between the set of states $\{|\nu\rangle\}$ — the solutions of Eq. (8) — and the set of states that solve $a_Q(\mathbf{k})|n\rangle = 0$. We are able to exploit this unitary equivalence to explicitly construct the states in $\{|\nu\rangle\}$, and to reformulate QED as a theory of charged particles that obey Gauss's law — that therefore carry their Coulomb field with them — and that interact with each other and with transversely polarized propagating photons [2,3]. In YM theory and QCD, however, the commutation rules among the components of $\Omega^a(\mathbf{k})$ are quite different from the commutation rules among the corresponding $a^a_Q(\mathbf{k})$. The latter 'ghost' annihilation operators commute with each other and with all their conjugate $a^{a\star}_Q(\mathbf{k})$. However, the commutation rules for $\Omega^a(\mathbf{k})$ and $\Omega^{a\star}(\mathbf{k})$ follow the $SU(N)$ Lie algebra [1].

[1] $\Omega^a(\mathbf{k})$ — and $\Omega^{a\star}(\mathbf{k})$ — in this work and in Ref. [1] differ by a normalization factor of $2k^{3/2}$.

It is therefore impossible to construct a unitary transformation that transforms $\Omega^a(\mathbf{k})$ into $a_Q^a(\mathbf{k})$. This difference between QED on the one hand and YM theory and QCD on the other, precludes the use of unitary transformations to construct the set of physical states $\{|\nu\rangle\}$ from the perturbative states $\{|n\rangle\}$ in the non-Abelian gauge theories; and that fact accounts for major differences between QED and the non-Abelian YM theory and QCD.

In formulating a procedure for constructing the state $|\nu_0\rangle$ in non-Abelian gauge theories, we note that whereas $\Omega^a(\mathbf{k})|\nu\rangle = 0$ and $\Omega^{a\star}(\mathbf{k})|\nu\rangle = 0$ are two wholly independent conditions for QED — and the former alone suffices to define the Fock space for this Abelian theory — these two conditions are not independent for YM theory and QCD. In YM theory and QCD, Eq. (8) requires that $\Omega^{a\star}(\mathbf{k})|\nu\rangle = 0$ too [1,5,6]. The appropriate condition for imposing Gauss's law in YM theory therefore is not Eq. (8), but

$$[\Omega^a(\mathbf{k}) + \Omega^{b\star}(-\mathbf{k})]|\nu\rangle = 0 \ . \tag{9}$$

We will express this condition as

$$[b_Q^a(\mathbf{k}) + J_0^a(\mathbf{k})]|\nu\rangle = 0 \ , \tag{10}$$

where we define

$$b_Q^a(\mathbf{k}) = k^{\frac{3}{2}}[a_Q^a(\mathbf{k}) + a_Q^{a\star}(-\mathbf{k})] \ . \tag{11}$$

We will transform the perturbative vacuum state $|0\rangle$ — the state that is annihilated by $a_s^a(\mathbf{k})$, $a_Q^a(\mathbf{k})$ and $a_R^a(\mathbf{k})$ — into a state $|\nu_0\rangle$ that implements Gauss's law. We represent $|\nu_0\rangle$ as a product of two operators acting on the perturbative vacuum $|0\rangle$, in the form

$$|\nu_0\rangle = \Psi \; \Xi \; |0\rangle \ , \tag{12}$$

where the operator product $\Psi \, \Xi$ represents a non-unitary transformation operator. We also define a state $|\phi_0\rangle = \Xi \; |0\rangle$, so that

$$b_Q^a(\mathbf{k})|\phi_0\rangle = 0 \ . \tag{13}$$

Eq. (13) is satisfied by $\Xi = \exp\{-\sum_{\boldsymbol{\kappa}} a_R^{c\star}(\boldsymbol{\kappa}) a_Q^{c\star}(-\boldsymbol{\kappa})\}$, as is confirmed by the observation that $[a_Q^a(\mathbf{k}),\, \Xi\,] = -\, a_Q^{a\star}(-\mathbf{k})\,\Xi$ and $[a_Q^{a\star}(\mathbf{k}),\, \Xi\,] = 0$. The resulting state $|\phi_0\rangle$ is not normalizable — it is essentially the "Fermi" vacuum state [7], which is not commonly used in QED, but reappears here in the non-Abelian theory.

The construction of Ψ involves solving the equation $\{b_Q^a(\mathbf{k}) + J_0^a(\mathbf{k})\}\,\Psi\,|\phi_0\rangle = \Psi b_Q^a(\mathbf{k})|\phi_0\rangle$, or equivalently,

$$[b_Q^a(\mathbf{k}),\, \Psi\,] = -J_0^a(\mathbf{k})\,\Psi + B_Q^a \ , \tag{14}$$

where B_Q^a represents any operator product with b_Q^a on its extreme right-hand side, so that $B_Q^a|\phi_0\rangle = 0$. Eq. (14) is an operator differential equation, in which the commutator plays

the role of a generalized derivative. We introduce the following notation for the constituent parts from which Ψ will be assembled:

$$a_i^\alpha(\mathbf{r}) = A_{Ti}^\alpha(\mathbf{r}) \; , \tag{15}$$

$$x_i^\alpha(\mathbf{r}) = A_{Li}^\alpha(\mathbf{r}) \; , \tag{16}$$

and

$$\mathcal{X}^\alpha(\mathbf{r}) = [\tfrac{\partial_i}{\partial^2} A_i^\alpha(\mathbf{r})] \; , \tag{17}$$

where, $[a_i^\alpha(\mathbf{r}) + x_i^\alpha(\mathbf{r})] = A_i^\alpha(\mathbf{r})$, and since $A_{Li}^\alpha(\mathbf{r}) = \partial_i[\frac{\partial_j}{\partial^2} A_j^\alpha(\mathbf{r})]\,,\; x_i^\alpha(\mathbf{r}) = \partial_i \mathcal{X}^\alpha(\mathbf{r})$. We also need to define the combination

$$\mathcal{Q}_{(\eta)i}^\beta(\mathbf{r}) = [a_i^\beta(\mathbf{r}) + \tfrac{\eta}{\eta+1} x_i^\beta(\mathbf{r})] \; , \tag{18}$$

with η integer-valued.

We will also use the preceding operators to form the following composite operators:

$$\psi_{(1)i}^\gamma(\mathbf{r}) = \; f^{\alpha\beta\gamma}\, \mathcal{X}^\alpha(\mathbf{r})\; \mathcal{Q}_{(1)i}^\beta(\mathbf{r}) = \; f^{\alpha\beta\gamma}\, \mathcal{X}^\alpha(\mathbf{r})\, [a_i^\beta(\mathbf{r}) + \tfrac{1}{2} x_i^\beta(\mathbf{r})] \; , \tag{19}$$

$$\psi_{(2)i}^\gamma(\mathbf{r}) = -f^{\alpha\beta b}\, f^{b\delta\gamma}\, \mathcal{X}^\alpha(\mathbf{r})\, \mathcal{Q}_{(2)i}^\beta(\mathbf{r})\, \mathcal{X}^\delta(\mathbf{r}) = -f^{\alpha\beta b}\, f^{b\delta\gamma}\, \mathcal{X}^\alpha(\mathbf{r})\, [a_i^\beta(\mathbf{r}) + \tfrac{2}{3} x_i^\beta(\mathbf{r})]\, \mathcal{X}^\delta(\mathbf{r}) \; , \tag{20}$$

and the general η-th order term

$$\psi_{(\eta)i}^\gamma(\mathbf{r}) = \; (-1)^{\eta-1}\, f_{(\eta)}^{\vec{\alpha}\beta\gamma}\, \mathcal{R}_{(\eta)}^{\vec{\alpha}}(\mathbf{r})\; \mathcal{Q}_{(\eta)i}^\beta(\mathbf{r}) \; , \tag{21}$$

in which

$$\mathcal{R}_{(\eta)}^{\vec{\alpha}}(\mathbf{r}) = \prod_{m=1}^{\eta} \mathcal{X}^{\alpha[m]}(\mathbf{r}) \; , \tag{22}$$

and

$$f_{(\eta)}^{\vec{\alpha}\beta\gamma} = f^{\alpha[1]\beta b[1]}\;\, f^{b[1]\alpha[2]b[2]}\, f^{b[2]\alpha[3]b[3]}\; \cdots\; f^{b[\eta-2]\alpha[\eta-1]b[\eta-1]} f^{b[\eta-1]\alpha[\eta]\gamma} \; , \tag{23}$$

where $f_{(1)}^{\vec{\alpha}\beta\gamma} \equiv f^{\alpha\beta\gamma}$.

We also define the composite operator

$$\mathcal{A}_1 = ig \int d\mathbf{r}\; \psi_{(1)i}^\gamma(\mathbf{r})\; \Pi_i^\gamma(\mathbf{r}) \; , \tag{24}$$

which is useful because it has the important property that its commutator with $b_Q^a(\mathbf{k})$,

$$\begin{aligned}[b_Q^a(\mathbf{k}),\, \mathcal{A}_1] = &-g\, f^{a\beta\gamma} \int d\mathbf{r}\; e^{-i\mathbf{k}\cdot\mathbf{r}}\; [a_i^\beta(\mathbf{r}) + x_i^\beta(\mathbf{r})]\; \Pi_i^\gamma(\mathbf{r}) \\ &-\tfrac{g}{2}\, f^{a\alpha\gamma} \int d\mathbf{r}\; e^{-i\mathbf{k}\cdot\mathbf{r}}\; \mathcal{X}^\alpha\; [\partial_i \Pi_i^\gamma(\mathbf{r})] \; ,\end{aligned} \tag{25}$$

generates $-J_0^a(\mathbf{k})$ when it acts on the "Fermi" vacuum state $|\phi_0\rangle$. We observe that

$$\partial_i \Pi_i^\gamma(\mathbf{r}) = \sum_{\mathbf{k}} b_Q^\gamma(\mathbf{k})\, e^{+i\mathbf{k}\cdot\mathbf{r}} \;, \tag{26}$$

so that $\partial_i \Pi_i^\gamma(\mathbf{r})|\phi_0\rangle = 0$, and

$$[b_Q^a(\mathbf{k}),\, \mathcal{A}_1]|\phi_0\rangle = -J_0^a(\mathbf{k})|\phi_0\rangle \;. \tag{27}$$

We might expect that the simple choice $\Psi = \Psi_0 = \exp(\mathcal{A}_1)$ would solve Eq. (14), but that expectation is not fulfilled. This is due to the fact that the commutator $[b_Q^a(\mathbf{k}),\, \mathcal{A}_1]$ does not commute with $\mathcal{A}_1$. The expression $\partial_i \Pi_i^\gamma(\mathbf{r})$, when it arises in the midst of an extended sequence of operator-valued factors, does not act on the Fermi vacuum state, and does not vanish. On its way to the extreme right of the expression, where it ultimately acts on $|\phi_0\rangle$ and vanishes, $\partial_i \Pi_i^\gamma(\mathbf{r})$ produces extra terms as it commutes with $\mathcal{A}_1$'s; and that fact disqualifies Ψ_0 as a solution of Eq. (14).

To address the problems that arise because $\mathcal{A}_1$ and $[b_Q^a(\mathbf{k}),\, \mathcal{A}_1]$ do not commute, we make the following modifications. First, we replace $\exp(\mathcal{A}_1)$ with $\|\exp(\mathcal{A}_1)\|$, where the $\|\mathcal{O}\|$ designates a variety of 'normal order' in which all functionals of momenta, $\mathcal{F}[\Pi_i]$, appear to the right of all functionals of gauge fields, $\mathcal{F}[A_i]$. For example, in the n^{th} term of $\|\exp(\mathcal{A}_1)\|$ the product $\|(\mathcal{A}_1)^n\|$ represents

$$\|(\mathcal{A}_1)^n\| = (ig)^n \int \mathsf{D}(1,\cdots,n)\, \psi^{\alpha_1}_{(1)i_1}(1)\; \psi^{\alpha_2}_{(1)i_2}(2)\; \cdots \psi^{\alpha_n}_{(1)i_n}(n)\; \Pi^{\alpha_1}_{i_1}(1)\; \Pi^{\alpha_2}_{i_2}(2)\; \cdots\; \Pi^{\alpha_n}_{i_n}(n) \;, \tag{28}$$

where $\mathsf{D}(1,\cdots,n)$ denotes $d\mathbf{r}_1 \cdots d\mathbf{r}_n$, and the integer argument n in the ψ's and Π's represents $\mathbf{r}_n$. This 'normal order' has the effect that, in $[b_Q^a(\mathbf{k}),\, \|\exp(\mathcal{A}_1)\|\,]$, the $\partial_i \Pi_i^\gamma(\mathbf{r})$ produced by an integration by parts, appears among the Π's and can annihilate the $|\phi_0\rangle$ directly, moving only through other Π's (or functionals of Π's) with which it commutes. The $(-g)\, f^{abc} \int d\mathbf{r}\, e^{-i\mathbf{k}\cdot\mathbf{r}}\, [a_i^b(\mathbf{r}) + x_i^b(\mathbf{r})]$ needed as part of $-J_0^a(\mathbf{k})$ is created to the left of $\|\exp(\mathcal{A}_1)\|$, as required; but the remaining $\Pi_i^c(\mathbf{r})$ appears to the right of all the ψ's, with which it *does not commute.* As $\Pi_i^c(\mathbf{r})$ moves to the left, to constitute the complete $-J_0^a(\mathbf{k})$, it generates unwanted contributions that disqualify even $\|\exp(\mathcal{A}_1)\|$ as a solution of Eq. (14). To compensate for the failure of $\|\exp(\mathcal{A}_1)\|$ to satisfy Eq. (14), we extend $\mathcal{A}_1$ so that it is only the first term in the infinite operator-valued series $\mathcal{A}$, given by $\mathcal{A} = \sum_{n=1}^{\infty} \mathcal{A}_n$, where the $\mathcal{A}_n$ with $n > 1$ will be given later in this section. Ψ is then given as

$$\Psi = \|\exp(\mathcal{A})\| \;. \tag{29}$$

All the $\mathcal{A}_n$ will consist of functionals of the gauge field, $A_i^\alpha(\mathbf{r})$, multiplied by a *single* momentum, $\Pi_i^\gamma(\mathbf{r})$, so that it becomes useful to express all the $\mathcal{A}_n$ as

$$\mathcal{A}_n = ig^n \int d\mathbf{r}\; \mathcal{A}^\gamma_{(n)i}(\mathbf{r})\; \Pi_i^\gamma(\mathbf{r}) \;. \tag{30}$$

Eq. (30) is useful as a definition of $\mathcal{A}^\gamma_{(n)i}(\mathbf{r})$, and as an identification of this quantity as a functional of gauge fields only — canonical momenta are never included in $\mathcal{A}^\gamma_{(n)i}(\mathbf{r})$.

In Eq. (14) we can now replace Ψ with Eq. (29) to give

$$[b_Q^a(\mathbf{k}),\, \|\, \exp(\mathcal{A})\, \|\,] \,+\, J_0^a(\mathbf{k}) \|\, \exp(\mathcal{A})\, \| \approx 0 \;, \tag{31}$$

where the symbol $\approx$ is used to indicate that we have suppressed the state $|\phi_0\rangle$, that should appear on the right of all operator products. We can expand Eq. (31) into the form

$$\begin{aligned} \|\, [b_Q^a(\mathbf{k}),\, \exp(\sum_{n=2}^{\infty} \mathcal{A}_n)\,]\, \exp(\mathcal{A}_1)\, \| \,+\, \|\, [b_Q^a(\mathbf{k}),\, \exp(\mathcal{A}_1)\,]\, \exp(\sum_{n=2}^{\infty} \mathcal{A}_n)\, \| \\ +\, J_0^a(\mathbf{k}) \|\, \exp(\mathcal{A})\, \| \approx 0 \;, \end{aligned} \tag{32}$$

where the $\|\quad\|$-ordering eliminates further contributions from the Baker-Hausdorff-Campbell formula. Since $[b_Q^a(\mathbf{k}),\, \|\, \exp(\mathcal{A})\, \|\,] = \|\, [b_Q^a(\mathbf{k}),\, \mathcal{A}\,]\, \exp(\mathcal{A})\, \|$, we can use Eq. (25) to eliminate $J_0^a(\mathbf{k}) \|\, \exp(\mathcal{A})\, \|$ and to rewrite Eq. (32) as

$$\|\, [b_Q^a(\mathbf{k}),\, \sum_{n=2}^{\infty} \mathcal{A}_n\,]\, \exp(\mathcal{A})\, \| \,-\, \|\, g\, f^{a\beta\gamma} \int d\mathbf{r}\, e^{-i\mathbf{k}\cdot\mathbf{r}} [a_i^\beta(\mathbf{r}) + x_i^\beta(\mathbf{r})]\, [\exp(\mathcal{A}),\, \Pi_i^\gamma(\mathbf{r})]\, \| \approx 0 \,. \tag{33}$$

We can also use $[\|\, \exp(\mathcal{A})\, \|,\, \Pi_i^\gamma(\mathbf{r})] = \|\, [\mathcal{A},\, \Pi_i^\gamma(\mathbf{r})]\, \exp(\mathcal{A})\, \|$, to give[2]

$$\|\, \{\, [b_Q^a(\mathbf{k}),\, \sum_{n=2}^{\infty} \mathcal{A}_n\,] \,-\, g\, f^{a\beta\gamma} \int d\mathbf{r}\, e^{-i\mathbf{k}\cdot\mathbf{r}} [a_i^\beta(\mathbf{r}) + x_i^\beta(\mathbf{r})]\, [\mathcal{A},\, \Pi_i^\gamma(\mathbf{r}_1)]\, \}\, \exp(\mathcal{A})\, \| \approx 0 \,. \tag{34}$$

After simplifying Eq. (34) and expanding, we establish

$$[b_Q^a(\mathbf{k}),\, \sum_{n=2}^{\infty} \mathcal{A}_n\,] \,-\, g\, f^{a\beta\gamma} \int d\mathbf{r}\, e^{-i\mathbf{k}\cdot\mathbf{r}}\, [a_i^\beta(\mathbf{r}) + x_i^\beta(\mathbf{r})]\, [\sum_{n=1}^{\infty} \mathcal{A}_n,\, \Pi_i^\gamma(\mathbf{r})] \approx 0 \;, \tag{35}$$

as a sufficient condition for the validity of Eq. (34). We now rewrite the limit of the sum in the second term, $\sum_{n=1}^{\infty} \mathcal{A}_n \Rightarrow \sum_{n=2}^{\infty} \mathcal{A}_{n-1}$, to give

$$[b_Q^a(\mathbf{k}),\, \sum_{n=2}^{\infty} \mathcal{A}_n\,] \,-\, g\, f^{a\beta\gamma} \int d\mathbf{r}\, e^{-i\mathbf{k}\cdot\mathbf{r}}\, [a_i^\beta(\mathbf{r}_1) + x_i^\beta(\mathbf{r})]\, [\sum_{n=2}^{\infty} \mathcal{A}_{n-1},\, \Pi_i^\gamma(\mathbf{r})] \approx 0 \,. \tag{36}$$

Requiring Eq. (36) to hold for all values of g, we obtain the the recursion relation

$$[b_Q^a(\mathbf{k}),\, \mathcal{A}_n] \approx g\, f^{a\beta\gamma} \int d\mathbf{r}\, e^{-i\mathbf{k}\cdot\mathbf{r}}\, [a_i^\beta(\mathbf{r}) + x_i^\beta(\mathbf{r})]\, [\mathcal{A}_{n-1},\, \Pi_i^\gamma(\mathbf{r})] \;, \tag{37}$$

which holds for $n > 1$, because $\mathcal{A}_0 = 0$.

We have been able to construct the first six terms of the $\mathcal{A}$ series and to confirm their consistency with Eq. (37). Because of the structural regularity of $\mathcal{A}_1$ - $\mathcal{A}_6$, we can also infer the form of the general $\mathcal{A}_n$. The expressions for these $\mathcal{A}_n$ are most easily given in a partially

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[2]in an expression $\|\, [\omega, \xi\,]\, \zeta\, \|$, the commutator is always to be evaluated *before* the double-bar ordering is imposed.

recursive way, in terms of the $\mathcal{A}^{\gamma}_{(n)i}(\mathbf{r})$ previously defined in Eq. (30). The definition of each $\mathcal{A}_n$ (with $n > 1$) contains references to $\mathcal{A}^{\gamma}_{(n')i}(\mathbf{r})$ with $n' < n$, and, in turn, together with Eq. (30), defines the new $\mathcal{A}^{\gamma}_{(n)i}(\mathbf{r})$. The terms in the $\mathcal{A}$ series are given by Eq. (24) and by

$$\begin{aligned}\mathcal{A}_2 = {} & \tfrac{ig^2}{2}\int d\mathbf{r}\; \psi^{\gamma}_{(2)i}(\mathbf{r})\; \Pi^{\gamma}_i(\mathbf{r}) \\ & + ig^2\, f^{\alpha\beta\gamma}\int d\mathbf{r}\; \tfrac{\partial_j}{\partial^2}[\mathcal{A}^{\alpha}_{(1)j}(\mathbf{r})]\; a^{\beta}_i(\mathbf{r})\; \Pi^{\gamma}_i(\mathbf{r})\;,\end{aligned} \tag{38}$$

$$\begin{aligned}\mathcal{A}_3 = {} & \tfrac{ig^3}{3!}\int d\mathbf{r}\; \psi^{\gamma}_{(3)i}(\mathbf{r})\; \Pi^{\gamma}_i(\mathbf{r}) \\ & + ig^3\, f^{\alpha\beta\gamma}\int d\mathbf{r}\; \tfrac{\partial_j}{\partial^2}[\mathcal{A}^{\alpha}_{(2)j}(\mathbf{r})]\; a^{\beta}_i(\mathbf{r})\; \Pi^{\gamma}_i(\mathbf{r}) \\ & + ig^3\, f^{\alpha\beta\gamma}\int d\mathbf{r}\; \tfrac{\partial_j}{\partial^2}[\mathcal{A}^{\alpha}_{(1)j}(\mathbf{r})]\; [\delta_{ik} - \tfrac{1}{2}\tfrac{\partial_i\partial_k}{\partial^2}]\mathcal{A}^{\beta}_{(1)k}(\mathbf{r})\; \Pi^{\gamma}_i(\mathbf{r})\;,\end{aligned} \tag{39}$$

$$\begin{aligned}\mathcal{A}_4 = {} & \tfrac{ig^4}{4!}\int d\mathbf{r}\; \psi^{\gamma}_{(4)i}(\mathbf{r})\; \Pi^{\gamma}_i(\mathbf{r}) \\ & + ig^4\, f^{\alpha\beta\gamma}\int d\mathbf{r}\; \tfrac{\partial_j}{\partial^2}[\mathcal{A}^{\alpha}_{(3)j}(\mathbf{r})]\; a^{\beta}_i(\mathbf{r})\Pi^{\gamma}_i(\mathbf{r}) \\ & + ig^4\, f^{\alpha\beta\gamma}\int d\mathbf{r}\; \tfrac{\partial_j}{\partial^2}[\mathcal{A}^{\alpha}_{(2)j}(\mathbf{r})]\; [\delta_{ik} - \tfrac{1}{2}\tfrac{\partial_i\partial_k}{\partial^2}]\; \mathcal{A}^{\beta}_{(1)k}(\mathbf{r})\, \Pi^{\gamma}_i(\mathbf{r}) \\ & + ig^4\, f^{\alpha\beta\gamma}\int d\mathbf{r}\; \tfrac{\partial_j}{\partial^2}[\mathcal{A}^{\alpha}_{(1)j}(\mathbf{r})]\; [\delta_{ik} - \tfrac{1}{2}\tfrac{\partial_i\partial_k}{\partial^2}]\; \mathcal{A}^{\beta}_{(2)k}(\mathbf{r})\, \Pi^{\gamma}_i(\mathbf{r}) \\ & + \tfrac{ig^4}{2}\, f^{\alpha\beta b}\, f^{b\delta\gamma}\int d\mathbf{r}\; \tfrac{\partial_j}{\partial^2}[\mathcal{A}^{\alpha}_{(1)j}(\mathbf{r})]\tfrac{\partial_k}{\partial^2}[\mathcal{A}^{\delta}_{(1)k}(\mathbf{r})]\; a^{\beta}_i(\mathbf{r})\, \Pi^{\gamma}_i(\mathbf{r})\;,\end{aligned} \tag{40}$$

$$\begin{aligned}\mathcal{A}_5 = {} & \tfrac{ig^5}{5!}\int d\mathbf{r}\; \psi^{\gamma}_{(5)i}(\mathbf{r})\; \Pi^{\gamma}_i(\mathbf{r}) \\ & + ig^5\, f^{\alpha\beta\gamma}\int d\mathbf{r}\; \tfrac{\partial_j}{\partial^2}[\mathcal{A}^{\alpha}_{(4)j}(\mathbf{r})]\; a^{\beta}_i(\mathbf{r})\Pi^{\gamma}_i(\mathbf{r}) \\ & + ig^5\, f^{\alpha\beta\gamma}\int d\mathbf{r}\; \tfrac{\partial_j}{\partial^2}[\mathcal{A}^{\alpha}_{(3)j}(\mathbf{r})]\; [\delta_{ik} - \tfrac{1}{2}\tfrac{\partial_i\partial_k}{\partial^2}]\; \mathcal{A}^{\beta}_{(1)k}(\mathbf{r})\, \Pi^{\gamma}_i(\mathbf{r}) \\ & + ig^5\, f^{\alpha\beta\gamma}\int d\mathbf{r}\; \tfrac{\partial_j}{\partial^2}[\mathcal{A}^{\alpha}_{(2)j}(\mathbf{r})]\; [\delta_{ik} - \tfrac{1}{2}\tfrac{\partial_i\partial_k}{\partial^2}]\; \mathcal{A}^{\beta}_{(2)k}(\mathbf{r})\, \Pi^{\gamma}_i(\mathbf{r}) \\ & + ig^5\, f^{\alpha\beta\gamma}\int d\mathbf{r}\; \tfrac{\partial_j}{\partial^2}[\mathcal{A}^{\alpha}_{(1)j}(\mathbf{r})]\; [\delta_{ik} - \tfrac{1}{2}\tfrac{\partial_i\partial_k}{\partial^2}]\; \mathcal{A}^{\beta}_{(3)k}(\mathbf{r})\, \Pi^{\gamma}_i(\mathbf{r}) \\ & + \tfrac{ig^5}{2}\, f^{\alpha\beta b}\, f^{b\delta\gamma}\int d\mathbf{r}\; \tfrac{\partial_j}{\partial^2}[\mathcal{A}^{\alpha}_{(2)j}(\mathbf{r})]\tfrac{\partial_k}{\partial^2}[\mathcal{A}^{\delta}_{(1)k}(\mathbf{r})]\; a^{\beta}_i(\mathbf{r})\, \Pi^{\gamma}_i(\mathbf{r}) \\ & + \tfrac{ig^5}{2}\, f^{\alpha\beta b}\, f^{b\delta\gamma}\int d\mathbf{r}\; \tfrac{\partial_j}{\partial^2}[\mathcal{A}^{\alpha}_{(1)j}(\mathbf{r})]\tfrac{\partial_k}{\partial^2}[\mathcal{A}^{\delta}_{(2)k}(\mathbf{r})]\; a^{\beta}_i(\mathbf{r})\, \Pi^{\gamma}_i(\mathbf{r}) \\ & + \tfrac{ig^5}{2}\, f^{\alpha\beta b}\, f^{b\delta\gamma}\int d\mathbf{r}\; \tfrac{\partial_j}{\partial^2}[\mathcal{A}^{\alpha}_{(1)j}(\mathbf{r})]\; \tfrac{\partial_k}{\partial^2}[\mathcal{A}^{\delta}_{(1)k}(\mathbf{r})]\; [\delta_{il} - \tfrac{2}{3}\tfrac{\partial_i\partial_l}{\partial^2}]\mathcal{A}^{\beta}_{(1)l}(\mathbf{r})\; \Pi^{\gamma}_i(\mathbf{r})\;,\end{aligned} \tag{41}$$

and

$$\begin{aligned}\mathcal{A}_6 = {} & \tfrac{ig^6}{6!}\int d\mathbf{r}\; \psi^{\gamma}_{(6)i}(\mathbf{r})\; \Pi^{\gamma}_i(\mathbf{r}) \\ & + ig^6\, f^{\alpha\beta\gamma}\int d\mathbf{r}\; \tfrac{\partial_j}{\partial^2}[\mathcal{A}^{\alpha}_{(5)j}(\mathbf{r})]\; a^{\beta}_i(\mathbf{r})\Pi^{\gamma}_i(\mathbf{r})\end{aligned}$$

$$+ ig^6\, f^{\alpha\beta\gamma}\!\int\! d\mathbf{r}\ \tfrac{\partial_j}{\partial^2}[\mathcal{A}^{\alpha}_{(4)j}(\mathbf{r})]\,[\delta_{ik} - \tfrac{1}{2}\tfrac{\partial_i\partial_k}{\partial^2}]\ \mathcal{A}^{\beta}_{(1)k}(\mathbf{r})\,\Pi^{\gamma}_i(\mathbf{r})$$
$$+ ig^6\, f^{\alpha\beta\gamma}\!\int\! d\mathbf{r}\ \tfrac{\partial_j}{\partial^2}[\mathcal{A}^{\alpha}_{(3)j}(\mathbf{r})]\,[\delta_{ik} - \tfrac{1}{2}\tfrac{\partial_i\partial_k}{\partial^2}]\ \mathcal{A}^{\beta}_{(2)k}(\mathbf{r})\,\Pi^{\gamma}_i(\mathbf{r})$$
$$+ ig^6\, f^{\alpha\beta\gamma}\!\int\! d\mathbf{r}\ \tfrac{\partial_j}{\partial^2}[\mathcal{A}^{\alpha}_{(2)j}(\mathbf{r})]\,[\delta_{ik} - \tfrac{1}{2}\tfrac{\partial_i\partial_k}{\partial^2}]\ \mathcal{A}^{\beta}_{(3)k}(\mathbf{r})\,\Pi^{\gamma}_i(\mathbf{r})$$
$$+ ig^6\, f^{\alpha\beta\gamma}\!\int\! d\mathbf{r}\ \tfrac{\partial_j}{\partial^2}[\mathcal{A}^{\alpha}_{(1)j}(\mathbf{r})]\,[\delta_{ik} - \tfrac{1}{2}\tfrac{\partial_i\partial_k}{\partial^2}]\ \mathcal{A}^{\beta}_{(4)k}(\mathbf{r})\,\Pi^{\gamma}_i(\mathbf{r})$$
$$+ \tfrac{ig^6}{2}\, f^{\alpha\beta b}\, f^{b\delta\gamma}\!\int\! d\mathbf{r}\, \tfrac{\partial_j}{\partial^2}[\mathcal{A}^{\alpha}_{(3)j}(\mathbf{r})]\tfrac{\partial_k}{\partial^2}[\mathcal{A}^{\delta}_{(1)k}(\mathbf{r})]\ a^{\beta}_i(\mathbf{r})\,\Pi^{\gamma}_i(\mathbf{r})$$
$$+ \tfrac{ig^6}{2}\, f^{\alpha\beta b}\, f^{b\delta\gamma}\!\int\! d\mathbf{r}\, \tfrac{\partial_j}{\partial^2}[\mathcal{A}^{\alpha}_{(2)j}(\mathbf{r})]\tfrac{\partial_k}{\partial^2}[\mathcal{A}^{\delta}_{(2)k}(\mathbf{r})]\ a^{\beta}_i(\mathbf{r})\,\Pi^{\gamma}_i(\mathbf{r})$$
$$+ \tfrac{ig^6}{2}\, f^{\alpha\beta b}\, f^{b\delta\gamma}\!\int\! d\mathbf{r}\, \tfrac{\partial_j}{\partial^2}[\mathcal{A}^{\alpha}_{(1)j}(\mathbf{r})]\tfrac{\partial_k}{\partial^2}[\mathcal{A}^{\delta}_{(3)k}(\mathbf{r})]\ a^{\beta}_i(\mathbf{r})\,\Pi^{\gamma}_i(\mathbf{r})$$
$$+ \tfrac{ig^6}{2}\, f^{\alpha\beta b}\, f^{b\delta\gamma}\!\int\! d\mathbf{r}\ \tfrac{\partial_j}{\partial^2}[\mathcal{A}^{\alpha}_{(2)j}(\mathbf{r})]\ \tfrac{\partial_k}{\partial^2}[\mathcal{A}^{\delta}_{(1)k}(\mathbf{r})]\ [\delta_{il} - \tfrac{2}{3}\tfrac{\partial_i\partial_l}{\partial^2}]\mathcal{A}^{\beta}_{(1)l}(\mathbf{r})\ \Pi^{\gamma}_i(\mathbf{r})$$
$$+ \tfrac{ig^6}{2}\, f^{\alpha\beta b}\, f^{b\delta\gamma}\!\int\! d\mathbf{r}\ \tfrac{\partial_j}{\partial^2}[\mathcal{A}^{\alpha}_{(1)j}(\mathbf{r})]\ \tfrac{\partial_k}{\partial^2}[\mathcal{A}^{\delta}_{(2)k}(\mathbf{r})]\ [\delta_{il} - \tfrac{2}{3}\tfrac{\partial_i\partial_l}{\partial^2}]\mathcal{A}^{\beta}_{(1)l}(\mathbf{r})\ \Pi^{\gamma}_i(\mathbf{r})$$
$$+ \tfrac{ig^6}{2}\, f^{\alpha\beta b}\, f^{b\delta\gamma}\!\int\! d\mathbf{r}\ \tfrac{\partial_j}{\partial^2}[\mathcal{A}^{\alpha}_{(1)j}(\mathbf{r})]\ \tfrac{\partial_k}{\partial^2}[\mathcal{A}^{\delta}_{(1)k}(\mathbf{r})]\ [\delta_{il} - \tfrac{2}{3}\tfrac{\partial_i\partial_l}{\partial^2}]\mathcal{A}^{\beta}_{(2)l}(\mathbf{r})\ \Pi^{\gamma}_i(\mathbf{r})$$
$$+ \tfrac{ig^6}{3!}\, f^{\alpha\beta b}\, f^{b\mu c}\, f^{c\delta\gamma}\!\int\! d\mathbf{r}\ \tfrac{\partial_j}{\partial^2}[\mathcal{A}^{\alpha}_{(1)j}(\mathbf{r})]\ \tfrac{\partial_k}{\partial^2}[\mathcal{A}^{\mu}_{(1)k}(\mathbf{r})]\ \tfrac{\partial_l}{\partial^2}[\mathcal{A}^{\delta}_{(1)l}(\mathbf{r})]\ a^{\beta}_i(\mathbf{r})\ \Pi^{\gamma}_i(\mathbf{r})\ . \tag{42}$$

To arrive at a form for $\mathcal{A}_n$ for arbitrary n, it is convenient to define $\overline{\mathcal{A}^{\gamma}_i}(\mathbf{r})$ by Eq. (30) and by

$$\overline{\mathcal{A}^{\gamma}_i}(\mathbf{r}) = \sum_{n=1}^{\infty} g^n \mathcal{A}^{\gamma}_{(n)i}(\mathbf{r})\ , \tag{43}$$

so that

$$\mathcal{A} = i\!\int\! d\mathbf{r}\ \overline{\mathcal{A}^{\gamma}_i}(\mathbf{r})\ \Pi^{\gamma}_i(\mathbf{r})\ . \tag{44}$$

We also define

$$\overline{\mathcal{B}^{\beta}_{(\eta)i}}(\mathbf{r}) = \{a^{\beta}_i(\mathbf{r}) +\ [\delta_{ij} - \tfrac{\eta}{\eta+1}\tfrac{\partial_i\partial_j}{\partial^2}]\overline{\mathcal{A}^{\beta}_j}(\mathbf{r})\ \}\ , \tag{45}$$

and

$$\mathcal{M}^{\vec{\alpha}}_{(\eta)}(\mathbf{r}) \equiv \prod_{m=1}^{\eta} \tfrac{\partial_j}{\partial^2}[\overline{\mathcal{A}^{\alpha[m]}_j}(\mathbf{r})] = \tfrac{\partial_j}{\partial^2}[\overline{\mathcal{A}^{\alpha[1]}_j}(\mathbf{r})]\tfrac{\partial_l}{\partial^2}[\overline{\mathcal{A}^{\alpha[2]}_l}(\mathbf{r})]\cdots\tfrac{\partial_w}{\partial^2}[\overline{\mathcal{A}^{\alpha[\eta-1]}_w}(\mathbf{r})]\tfrac{\partial_k}{\partial^2}[\overline{\mathcal{A}^{\alpha[\eta]}_k}(\mathbf{r})]\,, \tag{46}$$

where η represents the same integer-valued parameter originally introduced in Eq. (18), that we will now observe to be necessary for matching each $\overline{\mathcal{B}^{\beta}_{(\eta)i}}(\mathbf{r})$ with its corresponding $\mathcal{M}^{\vec{\alpha}}_{(\eta)}(\mathbf{r})$ in the integral equation for $\overline{\mathcal{A}^{\gamma}_i}(\mathbf{r})$ given below. We now formulate this non-linear integral equation for $\overline{\mathcal{A}^{\alpha}_i}(\mathbf{r})$ as

$$\mathcal{A} = \sum_{\eta=1}^{\infty} \tfrac{ig^{\eta}}{\eta!}\!\int\! d\mathbf{r}\ \psi^{\gamma}_{(\eta)i}(\mathbf{r})\ \Pi^{\gamma}_i(\mathbf{r})$$

$$
\begin{aligned}
&+ig\, f^{\vec{\alpha}\beta\gamma}_{(1)} \int d\mathbf{r} \mathcal{M}^{\vec{\alpha}}_{(1)}(\mathbf{r})\, \overline{\mathcal{B}^{\beta}_{(1)i}}(\mathbf{r})\, \Pi^{\gamma}_i(\mathbf{r}) \\
&+\tfrac{ig^2}{2}\, f^{\vec{\alpha}\beta\gamma}_{(2)} \int d\mathbf{r}\, \mathcal{M}^{\vec{\alpha}}_{(2)}(\mathbf{r})\, \overline{\mathcal{B}^{\beta}_{(2)i}}(\mathbf{r})\, \Pi^{\gamma}_i(\mathbf{r}) \\
&+\tfrac{ig^3}{3!}\, f^{\vec{\alpha}\beta\gamma}_{(3)} \int d\mathbf{r}\, \mathcal{M}^{\vec{\alpha}}_{(3)}(\mathbf{r})\, \overline{\mathcal{B}^{\beta}_{(3)i}}(\mathbf{r})\, \Pi^{\gamma}_i(\mathbf{r}) \\
&+\cdots \\
&+\tfrac{ig^\eta}{\eta!}\, f^{\vec{\alpha}\beta\gamma}_{(\eta)} \int d\mathbf{r}\, \mathcal{M}^{\vec{\alpha}}_{(\eta)}(\mathbf{r})\, \overline{\mathcal{B}^{\beta}_{(\eta)i}}(\mathbf{r})\, \Pi^{\gamma}_i(\mathbf{r}) \\
&+\cdots \quad , \qquad\qquad (47)
\end{aligned}
$$

or, more succinctly,

$$
\mathcal{A} = \sum_{\eta=1}^{\infty} \tfrac{ig^\eta}{\eta!} \int d\mathbf{r}\, \{\, \psi^{\gamma}_{(\eta)i}(\mathbf{r}) \,+\, f^{\vec{\alpha}\beta\gamma}_{(\eta)} \int d\mathbf{r}\, \mathcal{M}^{\vec{\alpha}}_{(\eta)}(\mathbf{r})\, \overline{\mathcal{B}^{\beta}_{(\eta)i}}(\mathbf{r})\, \}\, \Pi^{\gamma}_i(\mathbf{r}) \,. \qquad (48)
$$

We observe that the leading terms of the perturbative solution of Eq. (47) — or equivalently Eq. (48) — agree with $\mathcal{A}_1$ - $\mathcal{A}_6$, the explicit forms given in Eqs. (24) and (38)-(42). We have confirmed that the entire perturbative series — $\mathcal{A}_n$ for arbitrary n — correctly satisfy the recursion relation given in Eq. (37); but we have not yet established that fact with complete rigor in so far as concerns its extension beyond $n = 6$. Previously in this paper, we have shown Eq. (37) to be a sufficient condition for the implementation of Gauss's law.

$|\nu_0\rangle$ is not the only state that implements Gauss's law. Any state $|\nu_{\mathbf{k}}\rangle = \Psi\,\Xi\, a^{a\dagger}_s(\mathbf{k})|0\rangle$ or $|\nu_{\mathbf{k}_1\cdots\mathbf{k}_i}\rangle = \Psi\,\Xi\, a^{a_1\dagger}_{s_1}(\mathbf{k}_1)\cdots a^{a_i\dagger}_{s_i}(\mathbf{k}_i)|0\rangle$, where $a^{a\dagger}_s(\mathbf{k})$ creates a transversely polarized gluon, is annihilated by the Gauss's law operator $\mathcal{G}$. The important question then arises: What changes occur in the S-matrix when the states $|\nu_{\mathbf{k}_1\cdots\mathbf{k}_i}\rangle$ are substituted for the perturbative states $|n_{\mathbf{k}_1\cdots\mathbf{k}_i}\rangle$ as incident and scattered states? We will not discuss this question in detail in this paper. But we observe that, while it has been shown that there is no change in S-matrix elements when the $|\nu\rangle$ and the corresponding $|n\rangle$ states that they replace are unitarily equivalent [2,3], in *this* case, in which the transformation Ψ is *not* unitary, there *will* be changes in the S-matrix elements when the $|\nu\rangle$ states are substituted for the $|n\rangle$ states. These changes, however, do not appear in the lowest order of perturbation theory, since the non-unitarity does not arise in the leading term of Ψ. The most interesting possibility, of course is that the contribution of Ψ as a whole can be evaluated, and the non-perturbative effect of Gauss's law on the S-matrix assessed.

Another question to be addressed deals with the non-perturbative solutions of the non-linear Eq. (47) or equivalently Eq. (48). Further work is required to clarify how these non-perturbative solutions are related to the gauge sectors connected by the large gauge transformations [8].

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